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Methods of Nonlinear Dynamics and Equilibrium Structures of Magnetoelastic Chains

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To study equilibrium structures of magnetoelastic chains we have introduced an equivalent system and examined the whole class of its solutions. Appearance of various structures of the chain is due to the choice of an appropriate minimizing solution of the equivalent dynamic system. Commensurate and incommensurate structures, transitions from ferromagnetic to antiferromagnetic states, and transitions to the states with alternating clusters of ordered spins are obtained. Conditions for appearance of chaotic structures and amorphous magnetic states of the chain are discussed.

KEY WORDS: Incommensurate structure; chaos; nonlinear resonance; magnetoelastic interaction.

1. INTRODUCTION

In the papers in Refs. 1–3 Dzyaloshinsky developed a theory of appearance of magnetic superstructures due to various kinds of interaction (relativistic exchange interaction of the "spin–spin" type, "spin–lattice" interaction, and strong anisotropic one). A magnetic superstructure was characterized by occurrence of a large period of modulation incommensurate with the main lattice period. Such a situation turned out, in essence, to be typical of the variety of physical systems (as an example, see Bak's review article article[4]). Extensive study was made of the common picture of the "commensurate–incommensurate" transition (*C–I*) in continuous models in the works of Pokrovsky.[5,6] Influence of elastic properties of the substrate was studied in Ref. 7. Study of discrete elastic chains led to some new physical effects due to the system's discreteness.[8–12] In particular, the

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occurrence of the structural disorder region appeared to be one of these important effects. This region always leads to blurring of the transition point (C–I) and its width is determined by the width of the stochastic layer that comes about in the region of destructing separatrix by weak perturbations. In cases studied in Refs. 10 and 11 the width of the "structural chaos" region is exponentially small.[13] Appearance of a second transition boundary and large metastable regions of structural disorder were considered in Ref. 12. Conditions for the existence of large regions of structural chaos were considered also in Refs. 8, 10, 15.

In accordance with these results one needs an exhaustive analysis of discrete models. The present paper is devoted to investigating a one-dimensional classical chain of spins with magnetoelastic interaction. In the absence of exchange interaction the atomic chain in question may undergo the (C–I) and (I–"chaos") transitions. The exchange interaction leads to influence of the atomic arrangement on mutual orientation of spins through the magnetoelastic interaction. This yields, in its turn, a variety of modulated magnetic structures, in particular, a chaotic magnetic one.

The difference of the considered model both from the cases of Refs. 1–3 and the model ANNNI[4] lies in the presence of the magnetoelastic links and in the existence of combined structural and magnetic transitions in the chain.

2. MODEL, EQUILIBRIUM CONDITIONS AND AN EQUIVALENT DYNAMIC SYSTEM

Consider a one-dimensional chain which is described by the Hamiltonian

$$H = \frac{1}{2}\sum_n p_n^2 + \frac{\alpha}{2}\sum_n (x_{n+1} - x_n - a)^2 + \sum_n [J_0 + \varepsilon(x_{n+1} - x_n - a)]\,\mathbf{S}_{n+1}\mathbf{S}_n + \sum_n V(x_n) \tag{2.1}$$

where p_n, x_n are the momentum and the coordinate of the nth atom, α is an elastic interaction constant, $\mathbf{S}_n$ is the spin of the nth atom (the spins of all atoms are assumed to be lying in one plane), J_0 and ε are the parameters of exchange and magnetoelastic interactions, a is a constant of the chain in the absence of all the interactions but the elastic one, V is an external field potential due to either a backing or molecular surrounding of the chain, etc. The field has a period a_0: $V(x + a_0) = V(x)$.

Equilibrium conditions for atomic chains are determined by the equations

$$\dot{p}_n = 0, \qquad \dot{\mathbf{S}}_n = 0$$

$$x_{n+1} - x_n - a + \frac{\varepsilon}{\alpha}\cos(\theta_{n+1} - \theta_n)$$

$$= x_n - x_{n-1} - a + \frac{\varepsilon}{\alpha}\cos(\theta_n - \theta_{n-1}) + \alpha^{-1}V'(x_n) \tag{2.2}$$

where θ_n is the angle formed by the spin $\mathbf{S}_n$ with the chain's axis. The condition $\dot{\mathbf{S}}_n = 0$ yields

$$[J_0 + \varepsilon(x_{n+1} - x_n - a)]\sin(\theta_{n+1} - \theta_n)$$
$$= [J_0 + \varepsilon(x_n - x_{n-1} - a)]\sin(\theta_n - \theta_{n-1}) = \text{const} \equiv g \tag{2.3}$$

Denote

$$I_{n+1} = x_{n+1} - x_n - a + \frac{\varepsilon}{\alpha}\cos(\theta_{n+1} - \theta_n) \qquad (x_{-1} = 0) \tag{2.4}$$

Equilibrium conditions (2.2) and (2.3) are reduced to the map-preserving measure

$$I_{n+1} = I_n + \alpha^{-1}V'(x_n), \qquad x_{n+1} = x_n + \omega(I_{n+1}) \tag{2.5}$$

where $\omega(I_{n+1})$ is found as the solution of Eq. (2.3)

$$[J_0 + \varepsilon(\omega(I_{n+1}) - a)]\sin(\theta_{n+1} - \theta_n) = g \tag{2.6}$$

or with regard to (2.4)

$$\frac{\varepsilon^2}{\alpha^2}\left\{1 - \frac{g^2}{[J_0 + \varepsilon(\omega(I_{n+1}) - a)]^2}\right\} = \{I_{n+1} - [\omega(I_{n+1}) - a]\}^2 \tag{2.7}$$

Now we consider the expression

$$\mathscr{H} = \mathscr{H}_0(I) - \alpha^{-1}V(x)\sum_{n=-\infty}^{\infty}\delta(z - n) \tag{2.8}$$

which will be termed hereafter a Hamiltonian of the equivalent dynamical system. It is easily seen then that if $d\mathscr{H}(I)/dI = \omega(I)$ and the "time" z corresponds to the ordinate along the axis of the chain, the corresponding Hamiltonian equations of motion

$$\dot{I} = -\frac{\partial\mathscr{H}}{\partial x} = \alpha^{-1}V'(x)\sum_{n=-\infty}^{\infty}\delta(z - n)$$
$$\dot{x} = \frac{\partial\mathscr{H}}{\partial I} = \frac{\partial\mathscr{H}_0}{\partial I} = \omega(I) \tag{2.9}$$

lead (after integration in the vicinity of the point $z = n$) exactly to the map (2.5). The point in (2.9) means differentiation with respect to z.

Thus the problem of determining the ground state of the N-atom chain with Hamiltonian (2.1) is reduced to the following one-particle dynamic problem. For the dynamic set (2.8) one should clear up all possible dynamic trajectories $(I(z), x(z))$ and with their help calculate H_N as a function of $(I(1), x(1); I(2), x(2); \ldots I(N), x(N))$. Then, minimization of the expression

$$F = \lim_{N \to \infty} \frac{1}{N} H_N \tag{2.10}$$

determines the minimizing trajectory $(\tilde{I}(z), \tilde{x}(z))$ and the ground state, respectively.

Map (2.5) indicates that H_N depends on two parameters $(I(1), x(0))$ as a function of displacements. Since the choice of the initial point $x(0)$ is not important, minimization occurs only over the initial value I_0. Moreover, spin variables lead to one more parameter g, in which one should minimize $F(g, I_0)$.

It is noteworthy that according to (2.9) the variables (I, x) are canonically conjugate and stand for action phase variables.

In a particular case for $V(x) = 0$ from (2.5) it follows that $I_{n+1} = I_n = \cdots = I_0$ and with the help of (2.2)–(2.5) we find

$$F = \frac{\alpha}{2}(\omega - a)^2 + \frac{\alpha}{\varepsilon}[J_0 + \varepsilon(\omega - a)](I_0 - \omega + a)$$
$$\omega = \omega(I_0, g) \tag{2.11}$$

Minimizing (2.9) over I_0 and g yields

$$\begin{aligned} I_0 = 0: \quad & x_{n+1} - x_n = a + \varepsilon/\alpha \\ & \theta_{n+1} = \theta_n + n\pi \end{aligned} \Bigg\} \quad (J_0 > 0)$$
$$\begin{aligned} g = 0: \quad & x_{n+1} - x_n = a - \varepsilon/\alpha \\ & \theta_{n+1} = \theta_n = \cdots = \theta_0 \end{aligned} \Bigg\} \quad (J_0 < 0) \tag{2.12}$$

This corresponds to such a well-known result as appearance of deformation by an amount $\pm\varepsilon/\alpha$ because of the magnetoelastic interaction.

3. NONLINEAR RESONANCE AND INCOMMENSURATE PHASE

We consider the Hamiltonian of the equivalent dynamic system (2.7) using the method expounded in Refs. 12–13. Represent (2.7) as a Fourier series

$$\mathscr{H} = \mathscr{H}_0(I) - \alpha^{-1} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} V_k \exp[(kx/a_0 - mz)\, 2\pi i] \tag{3.1}$$

From (3.1) and (2.9) it follows that the resonances are possible under the condition

$$\dot{x} = \omega(I_{mk}) = a_0 \frac{m}{k} \tag{3.2}$$

where I_{mk} is the value of action which satisfies (3.2).

Inserting (3.2) into (2.6) we find

$$I_{mk} = a_0 \frac{m}{k} - a \pm \frac{\varepsilon}{a} \left[1 - \left(\frac{g}{J_0^{(r)}} \right)^2 \right]^{1/2} \tag{3.3}$$

where the value of $J_0^{(r)} = J_0 + \varepsilon(a_0(m/k) - a)$ has meaning of the value of effective exchange at exact resonance (3.2).

We further consider the effective Hamiltonian that describes the system dynamics (3.1) in the vicinity of an isolated (m, k) resonance. For this we present $\mathscr{H}_0(I)$ in the form of an expansion

$$\mathscr{H}_0(I) \simeq \mathscr{H}_0(I_{mk}) + \omega(I_{mk})(I - I_{mk}) + \tfrac{1}{2}\omega'(I_{mk})(I - I_{mk})^2$$

and make a canonical transformation eliminating the term linear in $(I - I_{mk})$ (this corresponds to transition into the rotating coordinate system) and preserve only the resonance term in the sum in (3.1). This gives[13]

$$\begin{aligned} &\mathscr{H}_{mk} = \tfrac{1}{2} k q_0 \omega'(I_{mk})(\Delta I)^2 - 2k q_0 V_k \alpha^{-1} \cos \psi \\ &q_0 = 2\pi/a_0, \qquad \Delta I = I - I_{mk}, \qquad \psi = q_0 kx - 2\pi mz \end{aligned} \tag{3.4}$$

where the prime indicates differentiation with respect to I. One can easily see that the variables $(\Delta I, \psi)$ are a canonically conjugate pair. The validity condition for Eq. (3.4) has the form[14,13]

$$K_{km} = |k^2 q_0^2 V_k \omega'(I_{mk})/\alpha| \ll 1 \tag{3.5}$$

which means the smallness of perturbation compared to nonlinearity.

Using the resonance condition (3.3) and Eq. (2.6), to define $\omega(I)$ one can obtain

$$1/\omega'(I_{mk}) = 1 \pm \varepsilon^2 g^2/\{\alpha(J_0^{(r)})^2[(J_0^{(r)})^2 - g^2]^{1/2}\} \tag{3.6}$$

Solutions of the system (3.4) are found from equations

$$\frac{d\,\Delta I}{dz} = -\frac{\partial \mathscr{H}_{mk}}{\partial \psi}, \qquad \frac{d\psi}{dz} = \frac{\partial \mathscr{H}_{mk}}{\partial\, \Delta I} \tag{3.7}$$

Determining these solutions and putting $z = n$, we get

$$x_n = \frac{m}{k} a_0 n + \frac{a_0}{2} + \frac{a_0}{\pi} \begin{cases} \arcsin[\kappa\, \mathrm{sn}(Q_{km} n, \kappa)], & \kappa \leqslant 1 \\ \mathrm{am}(\kappa Q_{km} n, 1/\kappa), & \kappa \geqslant 1 \end{cases}$$

$$I_n = \frac{m}{k} a_0 - a \pm \frac{\varepsilon}{\alpha}\left[1 - \frac{g^2}{(J_0^{(r)})^2}\right]^{1/2} + 2\kappa A_{km} \begin{cases} \mathrm{cn}(Q_{km} n, \kappa), & \kappa \leqslant 1 \\ \mathrm{dn}(\kappa Q_{km} n, 1/\kappa), & \kappa \geqslant 1 \end{cases} \tag{3.8}$$

where

$$\kappa^2 = \alpha(E + q_0 V_k/\alpha)/2q_0 V_k$$
$$Q_{km} = q_0[\omega'(I_{mk}) V_k/\alpha]^{1/2}, \qquad A_{km} = Q_{km}/q_0 \omega'(I_{mk}) \tag{3.9}$$

and E is an arbitrary constant which determines the system energy (3.4); sn, cn, dn, am are Jacobian elliptic functions.

Among all the solutions of (3.8) one should choose only those which minimize F under condition (2.6). For arbitrary κ substitution of (3.8) in (2.6) or (2.7) gives $g = 0$, i.e., $\sin(\theta_{n+1} - \theta_n) = 0$. Consequently, neighboring spins are either parallel or antiparallel.

From condition $\partial^2 F/\partial g^2 > 0$ one can easily get

$$\mathrm{sign}[\sin(\theta_{n+1} - \theta_n)] = -\mathrm{sign}\, J_{n+1}$$
$$J_{n+1} \equiv J_0 + \varepsilon(\omega(I_{n+1}) - a) \tag{3.10}$$

Thus, mutual orientation of neighboring spins is designated by the sign of effective exchange J_{n+1}. This means also that the form of magnetic configuration of the chain depends on the spaces between atoms.

Taking $g = 0$ into account and putting (3.8) into (2.10) we find

$$F = \frac{\alpha}{2}\left(\frac{m}{k} a_0 - a\right)^2 + 2(1 - \kappa^2)\, V_k \pm J_0^{(r)} + 2\kappa(\alpha V_k)^{1/2}\left(\frac{m}{k} a_0 - a \pm \frac{\varepsilon}{\alpha}\right)$$
$$\times \begin{Bmatrix} 0 \\ \pi/2K(1/\kappa) \end{Bmatrix} + 4\kappa^2 V_k \begin{Bmatrix} \kappa^{-2}[E(\kappa)/K(\kappa) - 1 + \kappa^2] \\ E(1/\kappa)/K(1/\kappa) \end{Bmatrix} \tag{3.11}$$

where $K(\kappa)$ and $E(\kappa)$ are full elliptic integrals. The upper line refers to the case $\kappa \leqslant 1$, and the lower one to the case $\kappa \geqslant 1$.

For $\kappa \leqslant 1$ minimization of (3.11) over κ yields $\kappa = 0$. In accord with (2.5) and (3.2) this corresponds to the exact resonance. The atoms are uniformly distributed with a period

$$x_{n+1} - x_n = \text{const} = a_0(m/k) \tag{3.12}$$

and with effective spin interaction

$$J_{n+1} = \text{const} = J_0^{(r)} = J_0 + \varepsilon a_0(m/k - a/a_0) \tag{3.13}$$

From here and (3.10) it follows that if $J_0^{(r)} < 0$ there occurs ferromagnetic spin ordering, while for $J_0^{(r)} > 0$ there is antiferromagnetic ordering.

We consider now the minimization of (3.11) for $\kappa \geqslant 1$. This leads to the equation

$$\frac{4}{\pi}\left(\frac{V_k}{\alpha}\right)^{1/2} \kappa E\left(\frac{1}{\kappa}\right) = a_0 \frac{m}{k} - a \pm \frac{\varepsilon}{\alpha} \tag{3.14}$$

which has a solution only for

$$\delta \equiv a_0 \frac{m}{k} - a \pm \frac{\varepsilon}{\alpha} \geqslant \delta_c \equiv \frac{4}{\pi}\left(\frac{V_k}{\alpha}\right)^{1/2} \tag{3.15}$$

Thus, for $\delta \leqslant \delta_c$ there exists the sole trivial solution $\kappa = 0$ with the period (3.12) corresponding to the so-called commensurate phase. For $\delta \geqslant \delta_c$ there arises the solution with the incommensurate phase for which κ is specified by (3.14) and the modulation of values x_n is determined by (3.8). $\kappa = 1$ or $\delta = \delta_c$ is the transition point. The period of structural modulation of the chain is found from the solution (3.8) and for $\kappa \geqslant 1$ is equal to

$$\lambda = 2K(1/\kappa)/q_0 \kappa \alpha^{-1/2} V_k^{1/2} \tag{3.16}$$

By $a^* = a - J_0/\varepsilon$ we denote such a distance $x_{n+1} - x_n$ between the coordinates of neighboring atoms for which the effective exchange J_{n+1} changes the sign according to (3.10). Assume hereafter for clearness that $J_0 < 0$, $\varepsilon > 0$ and $a_0(m/k) > a^*$. Then for $\kappa = 1$ from (3.8) we have

$$x_{n+1} - x_n = \omega(I_{n+1}) = a_0(m/k) - 2(V_k/\alpha)^{1/2} \operatorname{ch}^{-1}[q_0(n+1)(V_k/\alpha)^{1/2}] \tag{3.17}$$

Hence it follows that under the condition

$$a_0(m/k) - 2(V_k/\alpha)^{1/2} > a - J_0/\varepsilon = a^* \tag{3.18}$$

there comes about the region in which the magnetic order (ferro- or antiferromagnetic) is opposite to the magnetic order outside it. The center of the region has the coordinate $n = -1$ and its boundaries are found from the condition

$$(m/k)\, a_0 - 2(V_k/\alpha)^{1/2} \operatorname{ch}^{-1}[q_0(n+1)(V_k/\alpha)^{1/2}] = a^* \tag{3.19}$$

According to (3.19) and (3.15) the size of the region is equal to

$$\Delta n \sim 2n_0 \simeq \left(\frac{4a_0}{\pi^2 \delta_c}\right) \operatorname{ch}^{-1} \frac{\pi \varepsilon \alpha \delta_c}{2(\varepsilon \alpha \delta_c + \alpha J_0 + \varepsilon^2)} \tag{3.20}$$

In a general case, under the condition (3.5), solutions of the (3.8) type exist in the region[12–14]

$$\delta_c < \delta \lesssim 3\delta_c \tag{3.21}$$

For $\delta \gtrsim 3\delta_c$ perturbation of the periodic field $V(x)$ is small. It is a fast oscillating function and gives a contribution only to the order V^2. Hence it appears that for $\delta \gtrsim 3\delta_c$ the structural phase is determined with an accuracy up to V^2 by the uniform distribution of atoms with period (2.12). This distribution is modulated with the period a_0 of an external field and with amplitude $\sim V_k$. Elongation of the chain then equals zero with the accuracy up to the terms $\sim V_k^2$.

Let us find in the region (3.21) such a relation between the system parameters at which the effective exchange J_{n+1} vanishes. According to (3.10) we find

$$J_0 + \varepsilon[(m/k)\, a_0 - a + 2(V_k/\alpha)^{1/2} \kappa_1] = 0 \tag{3.22}$$

where κ_1 is the solution of Eq. (3.14). Equation (3.22), in particular, for $\kappa_1 - 1 \ll 1$, specifies the boundary

$$\delta_1 \simeq 2(\alpha J_0 + \varepsilon^2)/\alpha\varepsilon \geqslant \delta_c \tag{3.23}$$

which is a starting point of striped atomic structure with two types of magnetic order. They take turns with the period $\lambda(\kappa_1)$ (3.16). Further increasing of κ leads to

$$x_{n+1} - x_n = (m/k)\, a_0 + 2\kappa(V_k/\alpha)^{1/2} \operatorname{dn}(\kappa Q_{km} \cdot (n+1), 1/\kappa) \leqslant a^* \tag{3.24}$$

for all n. Some value of κ_2 corresponds to the equality sign in (3.24) at $n = -1$. By means of (3.14) and (3.15) it defines the value $\delta_2 < 3\delta_c$.

Thus, structural states of the chain in the incommensurate phase induce the following magnetic structures (for instance, for $J_0 < 0$, $\varepsilon > 0$, $(m/k)\, a_0 > a^*$, see Fig. 1).

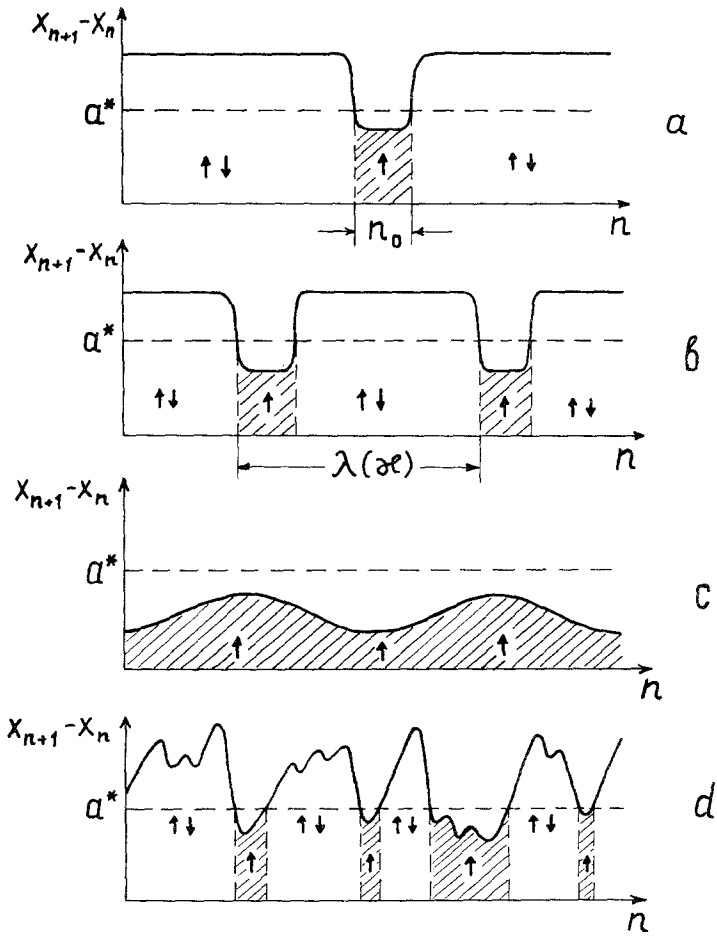

Fig. 1. Different equilibrium structures of the magnetoelastic chain: (a) the domain wall ($\kappa = 1$); (b) the ferromagnetic–antiferromagnetic alternating structure ($\kappa_1 < \kappa < \kappa_2$); (c) ferromagnetic incommensurate structure ($\kappa > \kappa_2$); (d) chaotic structure ($K > 1$). The ferromagnetic parts of the chain are cross-hatched.

1. There is antiferromagnetic ordering for $\delta < \delta_1$ (if the ordering will be ferromagnetic).

2. For $\delta_1 < \delta < \min(\delta_2, 3\delta_c)$ the bands of ferro- and antiferromagnetic order alternate. The summary size of two adjoining bands is equal to the period of the incommensurate structure λ.

3. For $\delta > 3\delta_c$ the resonance of the first order between the atomic chain and the field disappears and one may neglect magnetoelastic effects in the first order of V. The spin order then becomes ferromagnetic.

All the rest of the cases determined by the signs of the values J_0, ε, $(m/k)\,a_0 - a^*$ can be treated in a similar way.

4. STRUCTURAL CHAOS AND AMORPHOUS STATES OF THE CHAIN

The case of an isolated resonance (3.2) valid under (3.5) has been considered. Now we consider the chain's states in sufficiently strong fields $V(x)$, when instead of (3.5) and with regard to (3.6) and $g = 0$ the opposite condition

$$K_{km} = |k^2 q_0^2 V_k/\alpha| \gtrsim 1 \tag{4.1}$$

It is well known[14,16] that there is resonance overlapping in that case and stochastic "dynamics" of the equivalent dynamical system arise (2.8), (2.9).

If $K \ll 1$ the resonance interaction is small. In the vicinity of the separatrix ($\kappa = 1$) appears an exponentially narrow stochastic layer the width of which is equal in energy to[14]

$$\Delta\mathscr{H}_k = 32 V_k \exp(-\text{const}\, \alpha a_0/V_k^{1/2}) \tag{4.2}$$

where const ~ 1. This leads to blurring of the commensurate–incommensurate phase boundary of the structural transition point ($\kappa = 1$). Moreover, in the vicinity of this point stochastic fluctuations of atomic arrangement and, consequently, of the value of effective exchange J takes place. It is noteworthy that these fluctuations are due to a purely dynamical instability of the system.

One can observe quite another picture under condition (4.1). In that case from (2.5) issues the approximate local instability condition

$$\left| \frac{dx_{n+1}}{dx_n} - 1 \right| \sim \left| \frac{V''\omega'}{\alpha} \right| \gtrsim 1 \tag{4.3}$$

Using the expansion (3.1) and the definition of K in (4.1) we have the condition $K \cos\psi \gtrsim 1$. For sufficiently large K this means that there are small islands in phase space (I, ψ) in the vicinity of the points $\psi = \pi(2n + 1)/2$ in which motion is regular (not stochastic).[14,16,17] The island's size is $1/K$.

Minimum F realizes at $\psi = \pi/2$, i.e., in the center of the main island. However, even small fluctuations can "go through" the system into a large "stochastic sea" surrounding the islands. In that part of the phase space where the dynamics of the equivalent system are stochastic, the motion of the system takes place for a large enough time through a small probability to

return into the islands of small size. As a result one can regard all the system states corresponding to the stochastic dynamics as metastable states.[12] Random distribution of the atoms in the chain like in fluid corresponds to them. Thus, the amorphous magnetic structure arises. Here we merely outline some of its properties.

Under the condition $K \gg 1$ we know for the map like (2.5) the following correlation function[14-16]:

$$R_n = \langle\langle e^{iq_0 x_{n+m}} e^{-iq_0 x_m} \rangle\rangle = \frac{q_0}{2\pi} \int_0^{2\pi} dx_m \, e^{iq_0(x_{n+m} - x_m)}$$

$$\sim e^{-na/l_c} e^{in\omega(I_n)}, \qquad l_c = 2a/\ln K \equiv 2a/h \tag{4.4}$$

where the value h is the Kolmogorov entropy and the correlation decay length l_c determines the nearest order in the system. Uniform distribution in x establishes in the interval $(0, a)$. We shall proceed to study this case in a subsequent paper (see also Ref. 12).

In conclusion we should note the following. All the cases considered above were connected with the definition of the invariant curves of the map (2.5) for which condition (2.7) and the condition of smallness of H are the case. As shown in Refs. 18 and 19 for irrational values of $\omega(I)/a_0$ there exist, however, other invariant sets of the map (2.5). These sets correspond to a periodic trajectory; they have structure of Cantor type (cantori) and Hausdorf dimension <1. One can interpret them as the rest destroyed invariant curves in the Kolmogorov–Arnold–Moser theory. The stochastic trajectories pass through the gaps formed.

If the system is in the state, the corresponding periodic trajectory of which is cantorus and unattracting (through the Hamiltonian character of equations) then an arbitrarily small real perturbation will go through the system from this trajectory on the adjoining invariant curve. The role of such a perturbation can be played by small thermal fluctuations. In this connection the following question arises. Is it possible to observe the states corresponding to the cantori unattracting sets?

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